Math 173/273 - Spring, 2003 - Prof. Gu

Problem Set V - Due: 1:15 p.m., Monday, March 3

Do each of the following problems

(1) Find eigenvalues and corresponding eigenvectors for the following matrices over the field $\mathbb{C}$.

(a) \[
\begin{pmatrix}
2 & 4 \\
5 & 3
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
3 & 2 \\
-2 & 3
\end{pmatrix}
\]

(2) Find eigenvalues and corresponding eigenvectors for the following matrices over the field $\mathbb{R}$.

(a) \[
\begin{pmatrix}
5 & -6 & -6 \\
-1 & 4 & 2 \\
3 & -6 & -4
\end{pmatrix}
\]
(b) \[
\begin{pmatrix}
3 & 1 & -1 \\
2 & 2 & -1 \\
2 & 2 & 0
\end{pmatrix}
\]

(3) (a): Show that if $A$ and $B$ are similar, then $A$ and $B$ have same eigenvalues.

(b): Is the converse of (a) true? Justify your answer. (Hint: Exam problem 2 carefully.)

(4) Let $A_\phi$ be the $3 \times 3$ matrix representing a rotation of $\mathbb{R}^3$ through an angle $\phi$ about the y-axis.

(a): Find the eigenvalues for $A_\phi$ in the field $\mathbb{C}$.

(b): Determine necessary and sufficient conditions on $\phi$ in order for $A_\phi$ to have 3 linearly independent eigenvectors in $\mathbb{R}^3$. Justify your claim and interpret it geometrically.

(5) Let $A$ be a $2 \times 2$ matrix over $\mathbb{R}$ satisfying that $A^T = A$. Prove that $A$ has 2 linearly independent eigenvectors in $\mathbb{R}^2$.

(6) Exercise 1a, Chapter 6, Page 49 of Lax.

(7) For a vector $z = (z_1, z_2, \ldots, z_n) \in \mathbb{C}^n$, define the norm (or length) of $z$, denoted by

$$
||z|| = \sqrt{\sum_{i=1}^{n} |z_i|^2}.
$$

(Note: By the absolute value of a complex number $a+bi$, we mean $\sqrt{a^2 + b^2}$ and denote this by $|a+bi|$.) Do Exercise 1b, Page 49 of Lax, with the condition $A^N h \to \infty$ replaced by $\|A^N h\| \to \infty$.

(8) Exercise 2, Chapter 6, Page 53 of Lax.

(9) Exercise 3, Chapter 6, Page 53 of Lax.

Extra credit problem (Optional).

(1) On page 49, Exercise 1b), could you give a counter example of statement 1b)?

Reading Assignment: From page 45 of Lax to page 57 before Theorem 12.