First assume that $P$ is countably additive on the algebra $\mathcal{A}$. Now let $\{A_n\}$ be a sequence in $\mathcal{A}$ which decreases to the empty set. That is, $A_n \in \mathcal{A}$, $A_n \supset A_{n+1}$ for all $n$, and $\cap_{n=1}^{\infty} = \emptyset$. Then $A_n^c \uparrow \Omega \in \mathcal{A}$ as $n \to \infty$. Consequently, using the argument (or the result) of Theorem 1.5 on page 8, we see that $\lim_{n \to \infty} P(A_n^c) = P(\Omega) = 1$ and, therefore, $\lim_{n \to \infty} P(A_n) = 0$.

Now assume that whenever $A_n$ is a sequence in $\mathcal{A}$ which decreases to the empty set, then $\lim_{n \to \infty} P(A_n) = 0$. Suppose that $E_n$ is a sequence of mutually exclusive sets in $\mathcal{A}$ such that $E = \cup_{n=1}^{\infty} E_n \in \mathcal{A}$. Let $F_n = \cup_{k=1}^{n} E_k$ so that $F_n$ is a sequence in $\mathcal{A}$ that increases to $E$. So, if we let $A_n = E \cap F_n^c$, we get a sequence in $\mathcal{A}$ which decreases to the empty set. Therefore, according to our assumption, $\lim_{n \to \infty} P(A_n) = 0$. Since $P$ is finitely additive on $\mathcal{A}$ we know that $P(A_n) = P(E) - P(F_n)$ so that $\lim_{n \to \infty} P(F_n) = P(E)$. Again, by finite additivity, $P(F_n) = \sum_{k=1}^{n} P(E_k)$. Putting these pieces together we see that $P(E) = \lim_{n \to \infty} \sum_{k=1}^{n} P(E_k)$, i.e. $P(\cup_{n=1}^{\infty} E_n) = \sum_{n=1}^{\infty} P(E_n)$. Therefore, $P$ is countably additive on $\mathcal{A}$.