If we describe your opponent’s strategy by letting $x$ represent the first of her three successive results (either H or T), then her options reduce to the following 4 patterns: $xxx$, $xxy$, $xyx$, and $xyy$. So, you want to find the best strategy to counter each of these options and find the probability of winning in each of these situations.

1. $xxx$: Use $yxx$. In this case, the only way you can lose is if the first 3 tosses all produce an $x$. Otherwise, as long as a $y$ occurs among the first 3 tosses, the pattern $yxx$ must occur before the pattern $xxx$. So your probability of winning is $1 - 1/8 = 7/8$.

2. $xxy$: Again, use $yxx$. In this case, the only way you can lose is if the first 2 tosses each produce an $x$. Otherwise, as long as a $y$ occurs among the first 2 tosses, the pattern $yxx$ must occur before the pattern $xxy$. So your probability of winning is $1 - 1/4 = 3/4$.

3. $xyx$: Use $xxy$. Note that in this situation, nothing happens until the first $x$ appears. Let $p$ be the probability that your opponent wins from this position. There are two mutually exclusive ways this can take place. The first is that the next 2 tosses are $yx$, which has probability $1/4$. The second is that the next 2 tosses are $yy$, in which case nothing happens until the next $x$, and so this probability is $(1/4)p$. Note that if the next 2 tosses are either $xy$ or $xx$, she cannot win because either $xxy$ has appeared or $xxy$ will appear before $xyx$ in this case. Consequently, we have $p = 1/4 + (1/4)p$ which yields $p = 1/3$. So this strategy gives you a $2/3$ chance to win.

4. $xyy$: Again use $xxy$. This case is similar. Again, nothing happens until the first $x$ appears. Let $p$ be the probability that your opponent wins from this position. One way this can happen is if the next 3 tosses are $yy$ which has probability $1/4$. The other way is that the next 2 tosses are $yx$, in which case the process starts over again, so the probability is $(1/4)p$. Once more, if the next 2 tosses are either $xy$ or $xx$, she cannot win because either $xxy$ has appeared or $xxy$ will appear before $xyy$. Therefore, the same equation holds for $p$, namely $p = 1/4 + (1/4)p$, and thus $p = 1/3$. Using this strategy again produces a $2/3$ chance you will win.