Assignment Sheet#2

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Week of March 21, 2005: Measurable Functions (Random Variables) and Independence

• Reading: Read Chapter 3, noting that in class we have already given a different and more general proof of the assertion in Section 3.11, namely: the existence of a probability measure with a given distribution function. The results about properties of measurability, Section 3.2, and those on closure of the class of measurable functions with respect to a variety of operations, Sections 3.3 through 3.5, are very important. Another important section is 3.8 on the $\sigma$-algebra generated by a collection of functions. Sections 3.12 should be familiar, for example, from the Math 157 handout: “The Probability Transform and Simulation”. Sections 3.13 through (c) and 3.14 cover the more sophisticated material. Proofs of these important results are found in Appendix A.3. Continue reading in Chapter 4 through Section 4.8 on Markov chains. Section 4.7 on the existence of a sequence of independent random variables with prescribed distributions is very important.

• Problems to hand in: Due on Friday, April 1, 2005.

1. Let $(\Omega, F)$ be a measurable space and let $A \subset \Omega$. Show that $A$ is $\mathcal{F}$-measurable, i.e. $A \in \mathcal{F}$, iff its indicator function $I_A : \Omega \to \mathbb{R}$ is $\mathcal{F}$-measurable, i.e. measurable with respect to $\mathcal{F}$ and the Borel sets $\mathcal{B}$ of $\mathbb{R}$.

2. Suppose $(\Omega, \mathcal{F}, P)$ is a probability space and $A \in \mathcal{F}$. If we let $p = P(A)$ and $q = 1 - p = P(A^c)$, describe the distribution of the indicator random variable $I_A$ in terms of $p$ and $q$. Find another probability space with a random variable that has the same distribution.

3. A random variable on the measurable space $(\mathbb{R}^n, \mathcal{B}_n)$ is called a Borel function on $\mathbb{R}^n$. Show that every monotone function on $\mathbb{R}$ is a Borel function.

4. Exercise 3.13 (a) in Chapter 3.

5. Exercise E4.1 in Chapter E.

6. Exercise E4.2 in Chapter E.