Math 164 - HW 6

HW 6 Assignment to be placed in Notebook, may be collected on 3/10/05. Call for Presentations begins immediately.

1. Find and Plot Fourier Sine Series for \( f : [0, 1] \to \mathbb{R} \) defined by:

   (a) \( f(x) = \begin{cases} 
   x & x < \frac{1}{2} \\
   1 - x & x \geq \frac{1}{2} 
   \end{cases} \).

   (b) \( f(x) = \sin(2\pi x) + \frac{1}{100} \sin(10\pi x) \).

   (c) \( f(x) = (x - \frac{1}{2})^2 \).

2. Using the inner product \( \langle u, v \rangle = \int_{-1}^{1} u(x)v(x) \, dx \) for the space \( L^2((-1, 1), \mathbb{R}) \) of square-integrable functions on the interval \((-1, 1)\),

   (a) Perform Gramm-Schmitt orthonormalization process on \( \{1, x, x^2, x^3, \ldots \} \).

   (b) Expand \( f : (-1, 1) \to \mathbb{R} \) defined by \( f(x) = e^x \) in terms of this basis, i.e., compute \( P_X f \) for \( X = \text{span}\{1, x, x^2, x^3, \ldots \} \).