Math 164 - Quiz Six

Attempt 2 of 4 Problems; use no notes or calculators.

In what follows, assume zero Dirichlet boundary conditions for the simplest heat and wave equation models \( u_t = u_{xx} \) and \( u_{tt} = u_{xx} \), respectively, with \( t > 0, x \in (0, 1) \).

1. Solve the Heat Equation exactly, given an initial temperature distribution \( \sin \pi x + \frac{1}{10} \sin 10\pi x \). You do not need to show the separation of variables steps if you know the form of the general solution.

2. Solve the Wave Equation exactly, given an initial displacement \( \sin \pi x \) with zero initial velocity. You do not need to show the separation of variables steps if you know the form of the general solution.
3. Write down matrices $A$ and $B$ so that the forward solver and backward solver for a single time step of the heat equation can be viewed as computing $Au^i = u^{i+1}$ and solving $Bu^{i+1} = u^i$, respectively.

4. Given an initial displacement $f$ and an initial velocity $g$, explain the two methods for computing the displacement $u^1$ at the first time slice for the wave equation.