READ Problem D. Let \( u \) be an upper bound of non-empty set \( A \) in \( \mathbb{R} \). Prove that \( u \) is the supremum of \( A \) if and only if for all \( \epsilon > 0 \) there is an \( a \in A \) such that \( u - \epsilon < a \).

Note that to show that “\( S \) if and only if \( T \)” you must show that \( S \) implies \( T \), and \( T \) implies \( S \).

READ Problem E. Let \( A, B \) be nonempty subsets of \( \mathbb{R} \) that are bounded above, and let \( A + B = \{a + b : a \in A, b \in B\} \). Show that
\[
\sup(A + B) = \sup A + \sup B.
\]

Problem F. Let \( A, B \) be nonempty subsets of positive real numbers that are bounded above, and let \( A \cdot B = \{ab : a \in A, b \in B\} \). Show that
\[
\sup(A \cdot B) = \sup A \cdot \sup B.
\]

Problem G. (a) Let \( A \) be a nonempty subset of \( \mathbb{R} \) and suppose that \( s = \sup A \) belongs to \( A \). If \( b \) is not in \( A \), show that \( \sup(A \cup \{b\}) \) is equal to the larger of the two numbers \( s \) and \( b \).

(b) Use this to show that a nonempty finite set \( A \) contains its supremum. [Hint—use induction: show it is true first for a one-element set, then show that if it is true for an \( n \)-element set then it must be true for an \( (n + 1) \)-element set.]

Do also Chapter 1 (6ab, 6cd, 12, R13, 15).

Comment: When you are asked a question, e.g., problem 1.15, you should always give justification.