Math 132 — HW 5 — due Tuesday, October 3.

Do:
**Chapter 7** (18, 19) and **Chapter 8** (1).

And also:

A.) Find a solution to the following differential equation on \([0, 1]\) using the method below:

\[
\frac{df}{dx} = xf(x), \quad f(0) = 1. \quad (*)
\]

(a) Transform (*) into an integral equation.
(b) Construct an operator on functions

\[ T : C_b([0, 1]) \to C_b([0, 1]) \]

that is a contraction on \(C_b([0, 1])\) (verify this) and whose fixed point satisfies (*).
(Recall that the metric on \(C_b\) is \(d(f, g) = \|f - g\| = \sup_{[0,1]} |f(x) - g(x)|\).)
(c) Use \(T(f)\) and the method of successive approximations to solve (*) in terms of an infinite power series.
(d) What is the radius of convergence of your power series? Is your solution valid on more of \(\mathbb{R}\) than just \([0, 1]\)? Why?

B.) (a) Show that the radius of convergence \(R\) of the power series \(\sum c_n x^n\) is given by \(\lim_{n \to \infty} (|c_n|/|c_{n+1}|)\) provided this limit exists.
(b) Give an example of a power series for which the limit does not exist.

C.) What is the radius of convergence of \(\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n\)?

D.) In a few sentences, describe the main ideas in the course so far. What concepts would you test if you were to, say, write a midterm for this course?