**Some Preparatory Problems to ponder in advance**

1. The median of a data set of real numbers a number for which half the data lies on either side. But what if the data set are points in the plane? Can one define a kind of “median”? For instance, is there always a point in the plane for which any line through that point cuts the data set in half? Is there always a point so that any line through that point has at least 1/3 of the points on each side? Try some examples.

2. An art gallery is in a polygonal room (with finitely many sides, and possibly non-convex) and there is one painting hung on each wall. Suppose that for any 3 paintings, there is a point in the gallery where those 3 paintings are visible. Is there always a point of the gallery from which all paintings are visible?

3. Can a convex bounded polyhedron have seven edges?

4. Let $S$ be the unit square $[-1,1] \times [-1,1]$. Let $S^\Delta$ be the set of all points in the plane whose dot product with any point in $S$ is less than or equal to 1: $S^\Delta = \{ y \in \mathbb{R}^2 : y \cdot x \leq 1 \ \forall x \in S \}$. Find $S^\Delta$. Find $(S^\Delta)^\Delta$. See what this operation does to other sets $S$.

5. Take an $n$-gon and subdivide it into triangles whose vertices are chosen from the vertices of the $n$-gon. In this subdivision, how many triangles are needed to cover the $n$-gon? And in how many ways can this subdivision be done? (Find a formula in terms of $n$.)

6. Suppose you try to subdivide a cube into several tetrahedra, each of whose vertices are vertices of the cube. What is the minimum and maximum number of tetrahedra that will accomplish this? Prove your claims.

7. A cake (with some goodies on it) is divided by parallel knives into 3 pieces from left to right. This division can be represented by a vector $(x_1, x_2, x_3)$ where $x_i$ is the width of the $i$-th piece. Thus $x_i \geq 0$ and $\sum x_i = 1$, so that the set of all divisions forms a triangle (the piece of the plane $\sum x_i = 1$ in the first octant). Make whatever assumptions seem reasonable for your preferences (note that your preferences don’t have to be related to the width of the pieces). Let $C_i$ be the set of divisions on which you will prefer to take piece $i$. What properties will the sets $C_i$ satisfy? Must all the sets have a common point? Investigate the analogous problem for $n$ pieces.

8. Suppose you have graph formed by an $m \times n$ grid of squares, with each node (the places where the corners meet) labelled by either $+1$, $-1$, $+2$, or $-2$. Suppose there are boundary conditions: the nodes on the top (respectively, bottom) side of the graph are labelled $+1$ (respectively $-1$), and all remaining nodes on the right side (respectively, left side) are labelled $+2$ (respectively $-2$). Show that there must either be (i) an edge in the graph whose two labels sum to zero, or (ii) a square in the grid with all four labels.

9. Suppose a convex $n$-gon in the plane $P$ has integer vertex coordinates. Let $D(n)$ be the diameter of the smallest such $n$-gon. Try to find upper and lower bounds for $D(n)$.

10. Consider a new arithmetic on real numbers, in which $a \oplus b = \min(a, b)$ and $a \otimes b = a + b$. This gives corresponding operations on vectors, as well. Geometrically, one may then try to define lines and planes with the new arithmetic... how would you do that? Explore. What would it mean to find the root of a polynomial in this arithmetic?

11. Consider any variant of the above problems and investigate.