A possible list of topics for a 3-semester mathematics core

(prepared by J. Jacobsen, F. Su with help from other math faculty)
The order of these courses and order of topics might change, but this is a proof-of-concept for what a 3-semester core might look like.

Math 11 (assumes MWF schedule)

1. N,Z,Q,R,C & Induction (simple example)
2. Induction (more complicated examples), Proof by Contradiction (e.g., sqrt(2))
3. Limits (epsilon-delta) Part 1
4. Limits (epsilon-delta) Part 2 (proofs of limit theorems)
5. Differentiation (defn, proofs via definition of differentiation)
6. Proofs of Differentiation rules (sum, product, quotient, power for natural exponents)
7. Rolle’s Thrm, Mean Value Theorem & Applications (L’Hopitals Rule)
8. Integration (Riemann sums)
9. Proof of Fundamental Theorems of Calculus
10. Natural Log & its properties (proofs)
11. Exponential Function & its properties
12. Inverse Trig Functions, Integration by u-Substitution
13. Integration by Trig Substitution & Integration by Parts
14. Integration by Partial Fractions
15. The Complex Number System (arithmetic, polar form)
16. Complex limits and differentiation
17. Taylor polynomials and Taylor’s theorem (mention of Taylor series)
18. Infinite Series, Divergence Test, Geometric Series
19. Additional Convergence Tests (Ratio Test, Integral Test)
20. Absolute Convergence, Rearrangements, Complex Series, Radius of Convergence
21. Final Exam/Flex Day

Math 12 (assumes MWF)

1. vectors and dot products
2. matrices and properties
3. transposes and inverses
4. systems of equations, Gaussian elimination, echelon forms
5. examples, thinking in R^n, span, meaning of Ax=b
6. linear independence/dependence, homogenous/non-homog eqns
7. linear transformations, matrix of
8. invertibility, finding A-inverse
9. Invertible Matrix Theorem
10. subspaces of R^n, def'ns, spans are subspaces, null space
11. basis, dimension, unique rep'n of bases
12. row space, col space, bases for them, row rank=col rank, rank + nullity
13. determinants and volume
14. eigenvalues and eigenvectors
15. char eqn, properties, finding e-values and e-vectors

16. diagonalization and applications (dynamics of foxes/rabbits)
17. similar matrices and diagonalization
18. application: markov chains, google page rank

19. orthogonal sets, projections and matrices
20. orthogonal complements, Gram-Schmidt process
21. Spectral Theorem & Orthogonal Diagonalization of Symmetric Matrices

22. Fun: Merlin, review, flex day

**Math 13**

1. Intro to ODE’s, IVP’s, Classifications
2. First Order ODE’s I (e.g. linear)
3. First Order ODE’s II (e.g. separable)

4. Existence & Uniqueness
5. Modeling Example (e.g., pursuit curves, skydiver, etc.)
6. Second Order Linear Constant Coefficient, Real Roots

7. Complex Roots
8. Repeated Roots
9. Eulers’s Method

10. Driven ODE’s, Undetermined Coefficients
11. Linear Independence, Wronskian
12. Abel’s Theorem & Reduction of Order

13. Variation of Parameters
14. Forced Oscillations, Resonance
15. Practical Resonance

16. Intro to Systems, Compartment Models
17. Phase Plane
18. Linear systems and solution via eigendata

19. More linear systems
20. Nonlinear systems intro
21. Final Exam/ Flex Day

**Math 62**

1. Populations, Samples, Sample Space, Events, Axioms of Probability
2. Counting Techniques
3. Conditional Probability, Independence

4. Discrete Random Variables, Graphical Displays, Descriptive Statistics
5. Discrete Random Variables, Continuous Random Variables
6. Special Discrete Distributions
7. Continuous Random Variables, PDF, CDF, Expected Value and Variance
8. Normal and Gamma Distributions, Probability Plots
9. Jointly Distributed Random Variables, Covariance, Correlation
10. Sampling Distributions, Central Limit Theorem
11. Point Estimation
12. More methods of Point Estimation, Confidence Intervals
13. Confidence Intervals for Population Means and Proportions, Normal Populations
14. Hypothesis Testing
15. P-values, Statistical vs. Practical Significance
16. z-Tests, Two-sample t-Tests
17. Paired t-Test, Comparing population variances
18. Simple Regression Model, Parameter Estimation
19. Confidence Intervals for Regression, ANOVA preview
20. Interpreting Regression Results
21. Final Exam/Flex Day

Math 65 (Multivariable Analysis, semester course encompasses ideas from 14,61,63,64)

1. Lines, planes, dot product review, and cross product
2. Multivariable functions, graphing/visualizing
3. Limits of multivariable functions, continuity
4. Partial derivatives, tangent planes, differentiability/best linear approximation
5. derivative matrices, 2nd partials/meaning, mixed partials
6. Chain rule, directional derivatives
7. Gradient, properties
8. Parametrized paths, curves, velocity/acceleration, arclength
9. Vector fields/flow lines
10. Grad/Div/Curl, meaning, del operator
11. multivariable Taylor polynomials and optimization: max/min, hessian tests.
12. constrained optimization, Lagrange multipliers
13. Double integrals, meaning/properties, iterated integrals
14. Fubini, switching order of integration, type I/II/III regions
15. Triple integrals, meaning, choosing order of integration
16. Change of basis/Jacobian (emphasis on polar, cylindrical, spherical)
17. More cylindrical/spherical examples
18. Line integrals: scalar/vector, mass of wire, area of fence, work
19. gradient fields, conservative fields, fundamental theorem of line integrals
20. Nonconservative fields & Green’s Theorem
21. Surface/Flux integrals, symmetry arguments
22. Stokes’ theorem
23. Divergence Theorem, proof idea, applications
24. The Big Theorems (Green-Stokes/Divergence) and how they fit together
25. General Vector Spaces (especially function spaces)
26. Subspaces, Linear Independence, Dimension
27. Basis, Change of Basis

28. Matrix of a linear transformation
29. Inner Product Spaces, Norms, Distance Functions
30. Application (e.g., Least Squares Approximation, Fourier Approximation)

31. Systems of ODE’s
32. Existence & Uniqueness Theory for ODE system
33. Linear Systems & Solutions via Eigenvalues and Eigenvectors (2nd take)

34. Real Distinct Eigenvalues
35. Complex Eigenvalues, Degenerate Eigenspaces
36. Exponential of a Matrix

37. Driven Linear Systems
38. Variation of Parameters
39. Nonlinear Systems & Equilibrium Points

40. Linearization & Stability of Equilibria
41. Applications (predator-prey, pendulum, or similar topic of choice)
42. Final, Flex Day