Assignment # 5.
Due Wednesday, 18 April, in class.

Reading: Sections 3.1 (The Fatou and Julia Sets) and 3.3 (Normal Families and Equi-
continuity) of Beardon.

Recall that on problems marked (G) for ‘group’ you may cooperate, while on problems
marked (I) for ‘individual’ you may not cooperate and the only help you may get is from me.

Problems from Beardon:

Section 3.1, page 51 #2 (G), 3 (I), 4 (I).
Section 3.3, page 59 #2 (G).

Additional problem:

A. Continue preparation of your presentation. Make an appointment to meet with me to
discuss it.

Hint for Q2, p51. To prove that $D$ is in the Fatou set, use the definition of equi-
continuity. An $\varepsilon/2$ argument works for all sufficiently large $n$. To prove that every attracting
fixed point $p$ lies in the Fatou set, show that the iterates converge uniformly on some neigh-
bourhood of $p$ as follows. Show that $p$ has a neighbourhood on which $R(z)$ is a contraction:
there’s some $\lambda$ strictly less than 1 such that for all $z$ in the neighbourhood,
$$|R(z) - R(p)| \leq \lambda|z - p|.$$ To do this, start by showing that on the disc of convergence of the Taylor series for $R(z)$
about $p$, $R(z)$ can be written as
$$R(z) = R(p) + R'(z)(z - p) + (z - p)^2h(z),$$
where $h(z)$ is analytic and, on a closed subdisc of the disc of convergence, $h(z)$ is bounded.

A similar approach is useful for Q3, P51.